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Synthesis of Nonlinear Controller to Recover an Unstable Aircraft from Poststall Regime

Amitabh Saraf* and Girish Deodhare*
Centre for A.I. and Robotics, Bangalore 560 001, India
and
Debasish Ghose†
Indian Institute of Science, Bangalore 560 012, India

Dynamics of the aircraft configuration considered in this paper show a unique characteristic in that there are no stable attractors in the entire high angle-of-attack flight envelope. As a result, once the aircraft has departed from the normal flight regime, no standard technique can be applied to recover the aircraft. In this paper, using feedback linearization technique, a nonlinear controller is designed at high angles of attack, which is engaged after the aircraft departs from normal flight regime. This controller stabilizes the aircraft into a stable spin. Then a set of synthetic pilot inputs is applied to cause an automatic transition from the spin equilibrium to low angles of attack where the second controller is connected. This controller is a normal gain-scheduled controller designed to have a large domain of attraction at low angles of attack. It traps the aircraft into a low angle-of-attack level flight. This entire concept of recovery has been verified using six-degrees-of-freedom nonlinear simulation. Feedback linearization technique used to design a controller ensures internal stability only if the nonlinear plant has stable zero dynamics. Because zero dynamics depend on the selection of outputs, a new method of choosing outputs is described to obtain a plant that has stable zero dynamics. Certain important aspects pertaining to the implementation of a feedback linearization-based controller are also discussed.

Nomenclature

C_1	= controller at high angle of attack
C_2	= controller at low angle of attack
H	= height, m
P	= roll rate, deg/s
Q	= pitch rate, deg/s
R	= yaw rate, deg/s
V	= total velocity, m/s
α	= angle of attack, deg
β	= sideslip angle, deg
δe, δa, δr	= elevator, aileron, and rudder deflections, deg
θ	= pitch attitude, deg
ϕ	= bank angle, deg
ψ	= heading angle, deg
Ω	= aircraft rate of rotation about the velocity vector, deg/s

I. Introduction

POSTSTALL regimes of flight are characterized by the presence of highly nonlinear phenomena like wing rock, nose slice, and spin. It is important to understand aircraft dynamics in poststall regimes to design recovery methods. Bifurcation theory and continuation methods have played a significant role in understanding the poststall dynamics. Details of these methods and their application to aircraft dynamics can be found in Refs. 1–3 and references therein.

For the aircraft configuration studied in this paper, a comprehensive high angle-of-attack (AOA) aerodynamic database has been generated using rotary balance and forced oscillation wind tunnel data. Using this database and the aircraft dynamic model, a complete set of equilibrium points and limit cycles in poststall regime has also been obtained. Most of these equilibrium points and limit cycles are characterized by high yaw rates (>60 deg/s) and high AOA (>40 deg). Classically such types of limit sets are described as different types of spin.

The most notable feature of the dynamics of this aircraft configuration is that limit sets in the entire high AOA flight envelope are unstable. As a result the aircraft behavior in the high AOA flight regime is divergent and oscillatory. For illustration, a typical open-loop, i.e., with no controller connected, flight trajectory in the high AOA region is shown in Fig. 1. All of the important flight mechanics parameters shown in the figure illustrate divergence; e.g., note the divergence of α and sideslip angle β with time.

This situation is undesirable for two reasons. First, MIL-SPECS[4] require that the aircraft behavior beyond stall should be predictable and aircraft behavior in spin should be well defined. In typical spin recovery procedures, a pilot is instructed to wait until the poststall gyrations subside and the aircraft enters a well-defined spin. The pilot is then instructed to apply standard recovery methods to suppress yaw rate and reduce the high AOA for spin recovery. However, this standard recovery procedure cannot be adopted in the current aircraft because of the absence of a well-defined spin. Second, the current aerodynamic database is not valid for values of $\beta > 60$ deg and for highly oscillatory aircraft behavior because unsteady aerodynamics have been modeled to a very limited extent. Hence, divergent and oscillatory flight trajectories soon go beyond the valid database limits and, consequently, analysis of these trajectories is no longer valid. In Fig. 1 the simulation has stopped abruptly because the trajectory has gone beyond the valid database limit of $|\alpha| \leq 90$ deg.

It is not possible to design a control law at low AOA that can altogether prevent departures. It is also not possible to design a controller that has such a large domain of attraction that it can attract the aircraft from all high AOA conditions to a low AOA flight condition. This is because of several reasons such as the limitation of maximum control surface deflection and actuator power, the reversal of aileron effectiveness, and ineffective rudder at high AOA. In light of these difficulties, recovery of the current aircraft from a poststall situation to normal regime of flight requires a totally new methodology as suggested in this paper.

Two different control laws C_1 and C_2 are designed: one (C_1) for the high AOA nonlinear regime and the other (C_2) for the normal regime. The control law C_1 is designed to asymptotically stabilize the aircraft to a spin equilibrium condition at high AOA. A pilot initiates this control law upon realizing that the aircraft has gone past

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*Scientist, Controls Group; saraf/deodhare@cair.res.in.
†Associate Professor, Department of Aerospace Engineering; dghose@aero.iisc.ernet.in.

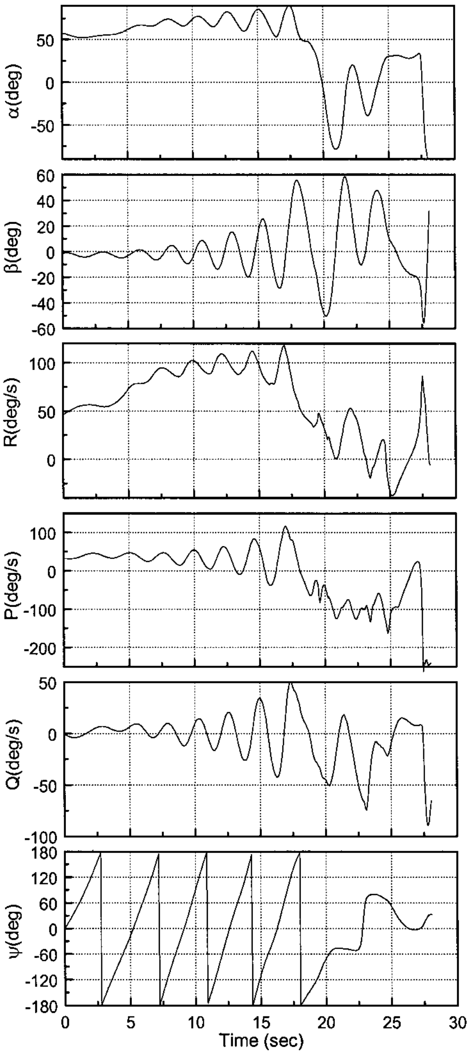

Fig. 1 Open-loop behavior of the unstable aircraft in high AOA regime of flight.

stall AOA and poststall gyrations have started. After taking over, the control law would asymptotically drive the aircraft to a preselected spin equilibrium. Once the aircraft is stabilized in spin, automatic recovery procedure is initiated. In this procedure, synthetic pilot inputs are designed that switch off C_1, suppress the yaw rate, and reduce AOA, and finally C_2 is switched on automatically once the yaw rate and AOA are made small enough to lie in the domain of attraction of C_2. Controller C_2 captures the aircraft in the low AOA regime of level flight.

Controller C_1 is designed using feedback linearization technique. To obtain a stable closed-loop system after feedback linearization, it is necessary that the nonlinear plant be minimum phase.[5,6] A new method is described in this paper to choose outputs that are linear combinations of aircraft states, so that the plant becomes minimum phase and hence stable after closing the loop. Some aspects of the implementation of this controller are also discussed and various simplifications are considered.

The paper is organized as follows. In Sec. II, the method of feedback linearization and the concept of zero dynamics are described. A new method to obtain stable zero dynamics is also described. In Sec. III, the controller C_1 that stabilizes a spin equilibrium is designed using the methods in Sec. II. A procedure to obtain the domain of attraction of C_1 is also described in this section. Section IV briefly describes the design of controller C_2 at low AOA. In Sec. V, design of synthetic inputs for automatic spin recovery are described. It is also explained why is it necessary to have both C_1 and C_2 and why C_2 alone is not enough to effect recovery. Section VI is devoted to the important issues of implementation and simplification of the controller C_1. The last section concludes the paper with discussions and some directions for future work.

II. Design of a Stabilizing Controller Using Feedback Linearization

The requirement for the controller C_1 is that it should stabilize an unstable spin equilibrium. Further, with the loop closed the stable spin equilibrium must have a reasonably large domain of attraction. Because the aircraft dynamics is quite nonlinear, a nonlinear controller design technique called feedback linearization[5–7] has been used to design C_1. There have been earlier attempts to apply this method to aircraft control design problem.[7–9] Reference 7 describes only the formulation of controller design using feedback linearization, and Refs. 8 and 9 present some closed-loop simulation results as well. In the preceding designs only low AOA database and flight envelope are considered. However, the real utility of the present method is in the poststall regimes of flight because the plant dynamics are much more nonlinear here than in the low AOA regimes.

A. Method of Feedback Linearization

Consider an affine nonlinear system with two inputs and two outputs; the inputs correspond to the elevator and aileron control surfaces:

$$\dot{x} = \boldsymbol{f}(x) + \boldsymbol{g}_1(x)u_1(t) + \boldsymbol{g}_2(x)u_2(t), \qquad y = Cx \tag{1}$$

where $x \in R^n$; u_1, $u_2 \in R^1$; $\boldsymbol{f}$, $\boldsymbol{g}_1$, $\boldsymbol{g}_2$ are smooth vector fields on R^n; and $C : R^n \to R^2$ is linear. The two control inputs are u_1 and u_2, and $y = [y_1 \;\; y_2]^T$ defines the two outputs of the system. Define $u = [u_1 \;\; u_2]^T$.

Now,

$$\dot{y} = C\boldsymbol{f}(x) + C\boldsymbol{g}_1(x)u_1 + C\boldsymbol{g}_2(x)u_2 \tag{2}$$

If the matrix

$$R(x) = [C\boldsymbol{g}_1(x) \quad C\boldsymbol{g}_2(x)] \tag{3}$$

is nonsingular, then it is possible to assign the control vector $\boldsymbol{u} = [u_1, \; u_2]^T$ as

$$\boldsymbol{u} = R(x)^{-1}v - R(x)^{-1}Cf(x) \tag{4}$$

where $\boldsymbol{v} = [v_1 \;\; v_2]^T$ is some external reference vector (in the case of an aircraft v_1 would be the pitch stick input and v_2 would be the roll stick input). The preceding nonlinear state feedback controller achieves the required input output feedback linearization because substitution of Eq. (4) into Eq. (2) yields

$$\dot{y} = \boldsymbol{v} \tag{5}$$

Thus the closed-loop system has two decoupled channels, and each channel output is an integral of the input. Using this controller, only two poles of the closed-loop linear system are placed at the origin. The remaining $(n - 2)$ modes constitute the zero dynamics of the system. For details, see Refs. 5–7.

B. Effect of Zero Dynamics on Internal Stability

The zero dynamics of a nonlinear system (1) can be briefly described as the unobservable part of the feedback linearized closed-loop system. Let x_0 be an equilibrium point of system (1) for some fixed u_1, u_2. If only the zero dynamics of the nonlinear system are linearized about x_0, then the eigenvalues of the linear system obtained are the same as the transmission zeros of the linear system[4] obtained by linearization of Eq. (1). This is a very useful result. The local asymptotic stability of a feedback linearized system depends on the stability of the unobservable modes of the closed loop, i.e., the zero dynamics of the open-loop plant. By the preceding result, it becomes quite easy to determine the local asymptotic stability of a feedback linearized system: If transmission zeros of the linear system obtained by linearization of Eq. (1) about x_0 are minimum phase, then the feedback linearized closed-loop system would be locally internally stable. In the next subsection, a new method is proposed to find a C matrix that can place transmission zeros at selected stable locations.

Note that feedback linearization is achieved via state feedback that depends on the outputs. However, the outputs that are chosen to stabilize the zero dynamics are only notional and not used for feedback. If only local stability of the closed loop is desired, any function of states can be chosen as outputs provided the zero dynamics for these outputs are stable.

C. Technique for Placement of Transmission Zeros of a Linear System

There have been many approaches to placement of transmission zeros in literature, e.g., see Refs. 10 and 11. In these methods the C matrix is assumed fixed and a feedthrough term is designed to place some of the zeros. In this paper, a method of placing transmission zeros by choosing a C matrix is described.

Consider a controllable system with n states and m inputs:

$$\dot{x} = Ax + Bu \tag{6}$$

It is required to find a linear map $C : R^n \to R^m$ such that the system [Eq. (6)] along with outputs defined by

$$y = Cx \tag{7}$$

has transmission zeros located at $Z = [z_1, z_2, \ldots, z_{n-m}]^T$. (Note that for a linear square system with n states, m inputs, and m outputs as defined in Eqs. (6) and (7) only $n - m$ finite transmission zeros exist.[11]) A constructive procedure to obtain a C matrix to place the zeros is described in the Appendix.

Remark. One problem with the method of zero placement described in the Appendix is that the outputs generally are a linear combination of all states. This could sometimes be undesirable because calculation of terms in $R(x)$ may become difficult. It requires considerable effort to obtain a C matrix with a given structure.

III. Design of Controller C_1: A Numerical Example

A. Selection of an Equilibrium Point and Linear Model

The first step in designing a nonlinear controller is to choose an equilibrium point for stabilization. An eighth-order system of equations has been used to model aircraft dynamics. The states consist of total velocity, AOA, angle of sideslip, roll rate, pitch rate, yaw rate, pitch attitude, and bank angle. Effect of altitude variation is neglected for the controller design. The values of the states at the chosen spin equilibrium points are $V = 80.4091$ m/s, $\alpha = 71.0519$ deg, $\beta = -1.5783$ deg, $P = 32.0513$ deg/s, $Q = -0.1358$ deg/s, $R = 93.0042$ deg/s, $\theta = -19.0159$ deg, and $\phi = -0.0837$ deg. The required control settings at this equilibrium point are

$$\delta e = -3.5 \text{ deg}, \qquad \delta a = 1.998 \text{ deg}, \qquad \delta r = 29.5 \text{ deg}$$

The equations of motion of the aircraft are first linearized about this equilibrium point, and the eighth-order linear model is given by

$$\dot{x} = Ax + Bu$$

where $x = [V, \alpha, \beta, P, Q, R, \theta, \phi]^T$, $u = [\delta e, \delta a]^T$,

$$B = \begin{bmatrix} -4.26964E{-2} & 5.62028E{-4} \\ -1.11845E{-4} & 0.00000E{+0} \\ -1.46300E{-5} & -2.53680E{-4} \\ -7.13970E{-5} & -4.71910E{-2} \\ -1.62045E{-2} & -4.62779E{-5} \\ -5.67266E{-6} & 3.70881E{-3} \\ 0.00000E{+0} & 0.00000E{+0} \\ 0.00000E{+0} & 0.00000E{+0} \end{bmatrix}$$

Because the rudder is completely masked by the wings and the fuselage, it cannot be used as a control input.

B. Placement of Zeros for C_1 Controller Design

In the preceding linear model, for eight states and two outputs, only six zeros can be placed. After some trial and error the final zero locations were chosen at $-0.1570 \pm 1.7558i$, $-0.3623 \pm 1.4290i$, $-0.1953 \pm 0.0481i$. For these locations, by using the preceding method, we obtain a simple C matrix:

$$C = \begin{bmatrix} 0 & 0.1 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

If the state feedback as in Eq. (4) were used, the remaining two eigenvalues would have been placed at the origin. To place these eigenvalues at any other location, say $\{-\lambda_1, -\lambda_2\}$, the state feedback needs to be modified to[6]

$$\begin{bmatrix} u_1(x) \\ u_2(x) \end{bmatrix} = (CB)^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - (CB)^{-1} \cdot \left[CAx + \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} Cx \right] \tag{9}$$

Using this expression, the other two eigenvalues have been placed at -2 and -3 to make the system more stable.

C. Implementation and Simulation

The preceding controller design requires only a linearized plant about the selected equilibrium point. In the nonlinear implementation of this controller in simulation, the plant should be represented in the affine form (1).

However, the equations of motion of an aircraft are not affine in the inputs because the moment and force contributions of the elevator and the aileron are complicated expressions involving states. Hence, to obtain the affine form, these moment and force contributions are linearized about their equilibrium values. The final form obtained after linearization is

$$\dot{x} = \tilde{F}(x) + \tilde{G}u \tag{10}$$

where $\tilde{F} : R^8 \to R^8$ and $\tilde{G} : R^2 \to R^8$ is a constant matrix and is not a function of states. The outputs are defined by $y = Cx$ with C as defined in Eq. (8). The complete expression of the feedback can be written by modifying Eq. (9) by replacing Ax with $\tilde{F}(x)$ and B with $\tilde{G}$.

$$A = \begin{bmatrix} -2.4359E{-1} & -4.6666E{+0} & -7.1899E{-1} & -1.2403E{-2} & -4.6166E{+0} & -2.9016E{-2} & 2.2313E{-2} & -2.4438E{-1} \\ -5.4267E{-4} & -1.4762E{-3} & -1.7242E{+0} & 8.74114E{-3} & 9.80341E{-1} & 2.55753E{-2} & 1.22060E{-1} & 4.71188E{-5} \\ -5.7792E{-5} & 1.72538E{+0} & -3.0967E{-2} & 9.44962E{-1} & -1.5113E{-3} & -3.2632E{-1} & -5.0466E{-5} & 1.15302E{-1} \\ -8.5704E{-5} & 9.78489E{-1} & -5.3852E{+0} & 1.11502E{-1} & -1.3360E{+0} & -3.6117E{-2} & 0.00000E{+0} & 0.00000E{+0} \\ -2.3628E{-2} & -2.1785E{+0} & -2.8697E{-1} & 1.52988E{+0} & -6.9998E{-1} & 6.41301E{-1} & 0.00000E{+0} & 0.00000E{+0} \\ 1.05584E{-3} & -4.6127E{-1} & -1.2780E{+0} & 1.26240E{-1} & -5.2224E{-1} & -9.6730E{-2} & 0.00000E{+0} & 0.00000E{+0} \\ 0.00000E{+0} & 0.00000E{+0} & 0.00000E{+0} & 0.00000E{+0} & 9.99998E{-1} & 1.46148E{-3} & 0.00000E{+0} & -1.6231E{+0} \\ 0.00000E{+0} & 0.00000E{+0} & 0.00000E{+0} & 9.99868E{-1} & 5.03612E{-4} & -3.4470E{-1} & 1.81611E{+0} & 1.50551E{-4} \end{bmatrix}$$

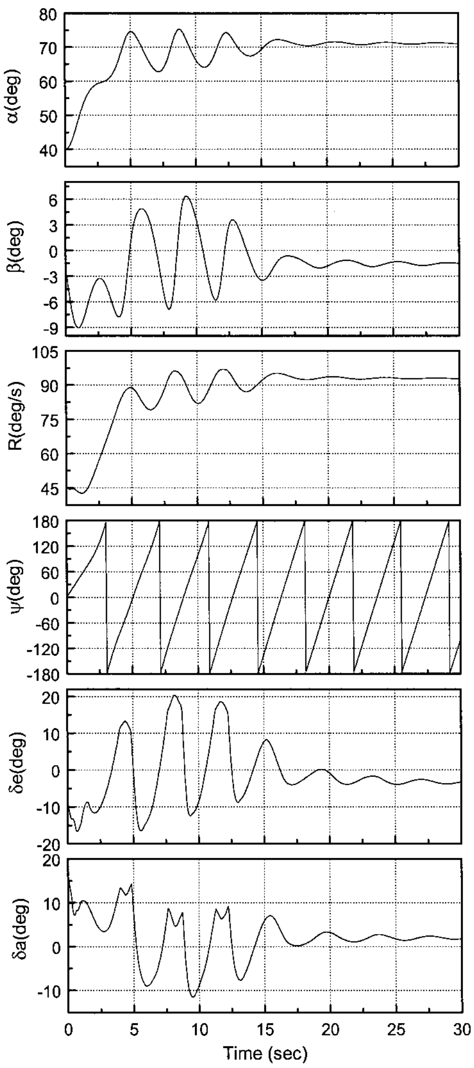

Fig. 2 Stabilization of a poststall condition by controller C_1.

This controller expression is coded in a six-degree-of-freedom nonlinear simulation and the closed-loop behavior is studied. The controller is implemented at a sampling rate of 80 Hz, and important hardware elements like the actuators along with their rate limits are also implemented in simulation.

Figure 2 shows the performance of the nonlinear controller. Here the controller C_1 is initiated at $t = 0$ s in a poststall condition that is quite far from the selected equilibrium point. In this simulation, the initial α is set to 40 deg and initial R is set to 45 deg/s. This condition corresponds to a typical situation just after stall when the poststall gyrations are starting. The elevator and aileron deflections shown in the figure are controller commands to the control surfaces that drive the aircraft to the equilibrium point in about 20 s. Note that no pilot inputs are required for this stabilization.

D. Estimation of the Domain of Attraction of C_1

It is very important to estimate the domain of attraction of the stable equilibrium with controller C_1. The stable spin at this equilibrium can be achieved only if the aircraft states at the time of engagement of C_1 lie within this domain. There are many methods available for estimating the domain of attraction of a stable equilibrium point for nonlinear systems.[12–14] However, it is very difficult to apply these methods to estimate the domain of attraction of an eighth-order closed-loop system because of the enormous computational effort required. As a result the domain of attraction is obtained using the method of simulations.

It is not possible to carry out simulations with every point in the state space as the initial condition. Experience has established that the two parameters required for characterization of spin are α and Ω.[1,15,16] These two variables along with β are also the parameters that are used to generate the present high AOA database. Hence, the domain of attraction is defined in a three-dimensional space of α, β, and Ω. Figure 3 shows the domain of attraction achieved by stabilizing the spin equilibrium by C_1. Each part of the figure shows the domain of attraction in α, Ω plane for different values of β. As can be seen, a fairly large region lies inside the domain of attraction. Note that only positive Ω is considered because C_1 can stabilize only the right-spin equilibrium. A similar controller needs to be designed for negative Ω that would stabilize a left-spin equilibrium. The pilot needs to make the judgment of selecting a proper controller based on the initial left-spin or right-spin tendency of the aircraft.

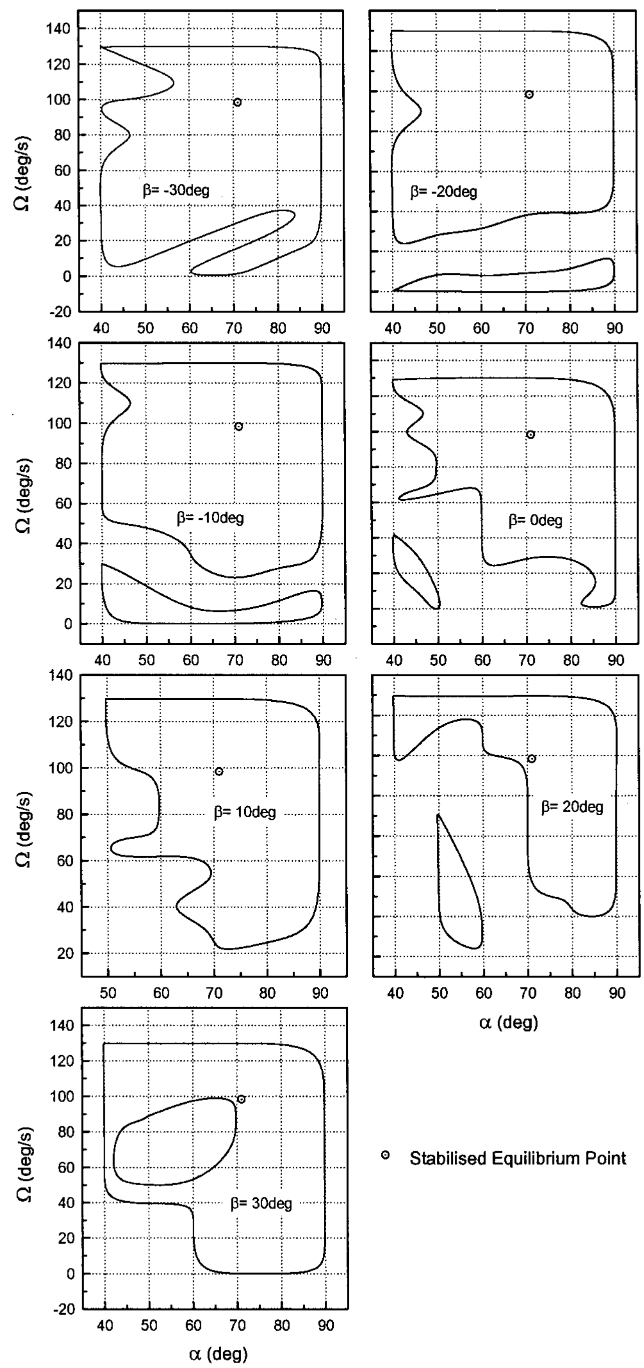

Fig. 3 Domain of attraction for the controller C_1 plotted for various sideslip angles.

IV. Design of Controller C_2

The requirement for controller C_2 is that it should have a sufficiently large domain of attraction at low AOA and Ω. Moreover, the closed-loop system should have a small time constant so that aircraft stabilizes to the low AOA condition in a short time. Presently, there exists a standard gain scheduled automatic flight control law (CLAW) that has been designed for stability augmentation and to achieve good handling qualities. CLAW has been designed using standard linear control design techniques with linear plant models for various flight conditions. The final form of CLAW is obtained by scheduling the gains and filter constants with Mach and altitude. The longitudinal feedback uses signals from pitch rate, α, and normal acceleration sensors, whereas the lateral-directional feedback uses signals from roll rate, β, yaw rate, and lateral acceleration sensors. The controller C_2 has been obtained by suitably modifying CLAW. The major modification required is the significant increase in the gains in the α and β feedback paths to increase the rate of convergence of trajectories to the low α equilibrium. The controller C_2 has a domain of attraction defined by α from -10 deg to about 30 deg. and Ω between -20 to 20 deg/s. This domain of attraction is significantly larger than the existing CLAW. The handling qualities of controller C_2 are not of interest because the pilot switches over to CLAW as soon as the aircraft stabilizes at low AOA.

V. Process of Spin Recovery

The manual procedure of spin recovery is fairly well established. After poststall gyrations have subsided and the aircraft has entered a well-defined spin, the pilot initiates recovery by giving an antispin roll stick to reduce the large yaw rate. When the yaw rate is reduced the pilot reduces the roll stick deflection and provides a nose-down pitch stick and reduces the AOA.

This technique of recovery does not work too well for this aircraft, as was established during the piloted spin recovery simulations on a flight simulator. It is difficult to decide in real time the exact timing for disengaging the controller C_1 and engaging the controller C_2. During the recovery procedure the aircraft may remain in the open-loop mode for some time. In this duration the aircraft becomes highly sensitive to pilot inputs, making it virtually impossible to control the aircraft during the transition from high AOA to low AOA. Hence, it is very difficult to recover the aircraft from the spin with the pilot in command.

To overcome this problem, an extensive study was carried out to generate synthetic pilot inputs that can effect recovery. After achieving a stable spin with the controller C_1 operating, the pilot can initiate recovery through a single recovery switch. Once switched on, synthetic inputs would be applied at pilot sticks and the engaging and disengaging of controllers would be carried out at precise timings automatically. A recovery sequence with the spin recovery switch is shown in Fig. 4. The points marked as A, B, C, D in the figure indicate various events that occur in the flight history. At $t = 0$ s, C_1 is switched on (marked as A in Fig. 4). At $t = 20$ s when the aircraft is stabilized in spin by C_1, the recovery switch is pressed (marked as B in Fig. 4). As a result, a synthetic antispin (negative) roll stick input is applied at a rate of 45 mm/s and yaw rate is monitored. At $t = 22.5$ s, as the yaw rate reduces to 55 deg/s, C_1 is automatically switched off (marked as C), the roll stick is brought to neutral at a rate of 20 mm/s, and α is monitored. Controller C_1 needs to be disengaged for recovery because C_1 strongly opposes recovery actions. When α reduces to about 30 deg at $t = 25$ s, C_2 is switched on (marked as D). The aircraft now achieves a constant α and zero yaw rate. The aircraft attains a wings-level stable flight starting from a poststall condition in about 32 s and with a loss of height of about 2.8 km.

It is often said that instability of spin equilibrium is a desirable feature because it aids recovery. An important assumption made in this statement is that there exists a stable equilibrium at low AOA that can attract and stabilize the aircraft. In the aircraft under study, there is no stable open-loop equilibrium at low AOA, nor can it be made very stable with CLAW operating. It has been seen that a combination of unstable spin equilibria and the stabilizing controller C_2 cannot effect recovery from all poststall conditions. Because of large gains in feedback paths, the control surfaces get saturated and the controller C_2 becomes ineffective in poststall conditions. In other words, the domain of attraction of C_2 is not large enough to effect recovery from all poststall conditions. The same conditions,

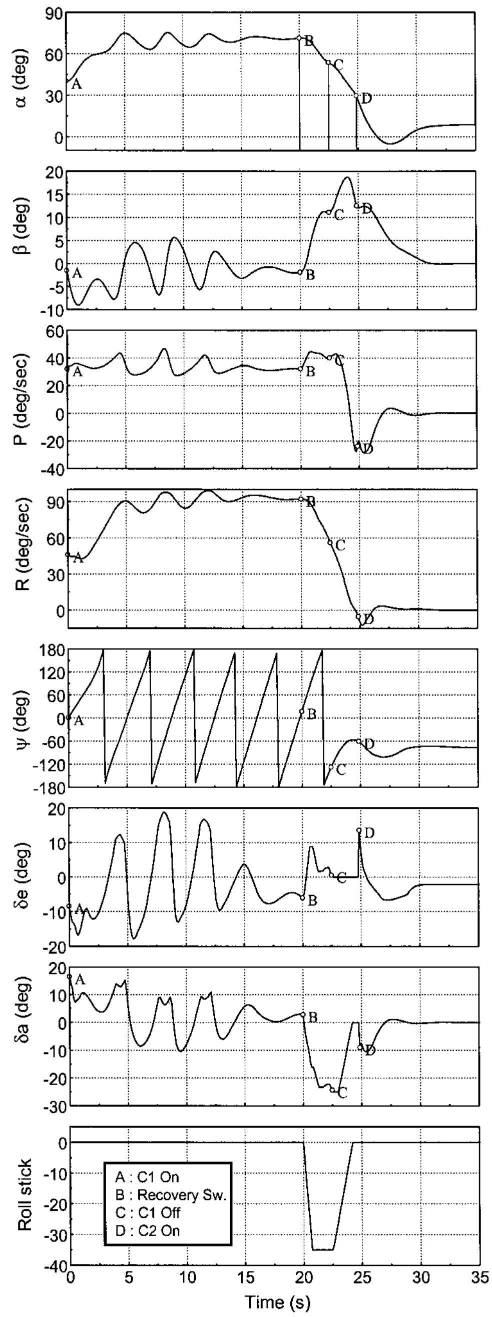

Fig. 4 Complete recovery sequence from a poststall condition, first stabilizing by C_1 to a spin equilibrium, then recovering using simulated inputs, and finally capturing at low α using C_2.

however, can be quite easily recovered by using the two-controller philosophy suggested in this paper.

VI. Possible Simplifications of the Controller for Implementation

It is necessary to consider implementation aspects of the controller C_1 because intensive computations are required to obtain function values $\tilde{F}(x)$ in Eq. (10). By studying the controller behavior it seems possible to simplify the controller in three possible ways: 1) by reducing the number of states used for feedback, 2) by approximating the nonlinear database, and 3) by function approximating the controller. Some of the results using the first and the second methods are described next.

A. Simplification by Reducing Number of States in Computation of Feedback

Feedback linearization-based controller is a state feedback controller and, hence, for each state x it is possible to determine the

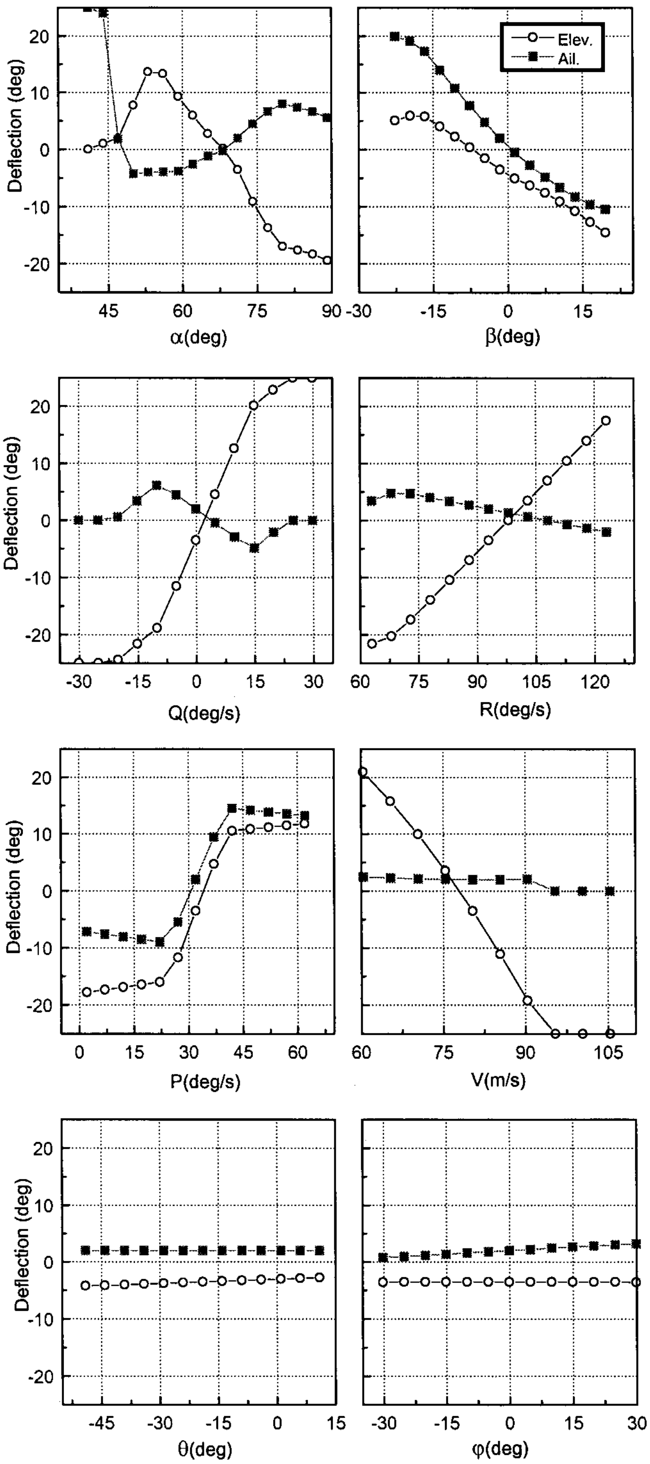

Fig. 5 Deflection of control surfaces with C_1 connected and states varied one at a time about the equilibrium point.

surface deflection that would be demanded by the controller. Figure 5 shows the aileron and elevator deflections demanded by the controller when only one of the states is varied and others are kept constant at their respective trim values. As can be seen, the surface deflections demanded for variation in pitch attitude and bank angle are very small. Hence it is possible to simplify the controller by not using θ and ϕ variation in the feedback. Figure 6 compares behavior of the simplified controller with the normal controller for an initial condition far away from the equilibrium point. The figure shows that aircraft can still be stabilized in approximately the same time, though the variations of states and the controller effort are more for the simplified controller.

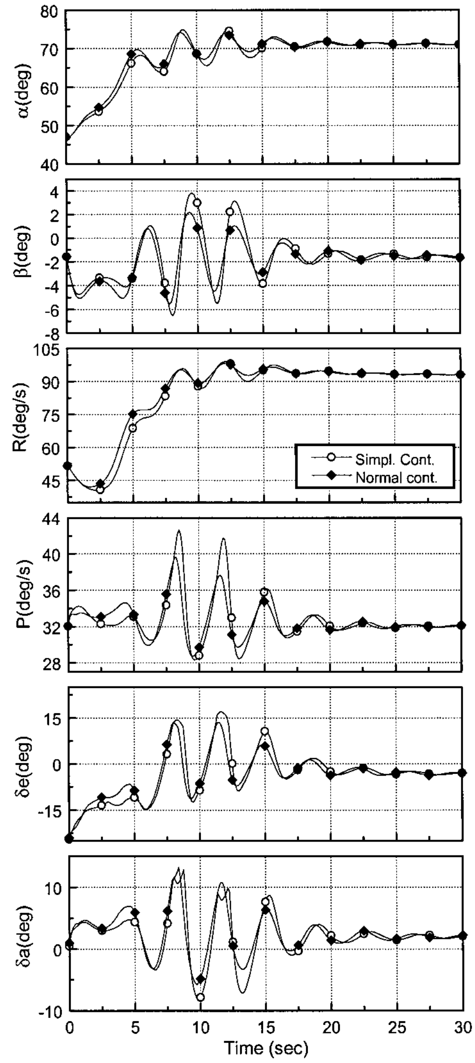

Fig. 6 Stabilization using the normal controller C_1 and the simplified controller that does not use θ and ϕ for feedback.

B. Simplification by Approximating the Database

The implementation of controller C_1 in its original form requires that the entire database be carried in the flight control computer. This is not acceptable because storing an aerodynamic database requires large memory on the computer and the extraction of aerodynamic derivatives is a significant computational burden. Hence it is useful to see if the coefficients can be represented as mathematical expressions in terms of the aircraft states. However, it is necessary to ensure that, with the new representation of the database used in controller calculation, the domain of attraction of the closed loop remains reasonably large.

The coefficients of forces and moments in the database being used are a function of the three rotation rates and α and β. As a simple representation, these functions can be approximated about their trim values and written as

$$
\begin{aligned}
C_i(\alpha, \beta, P, Q, R) &= C_{i_{\text{trim}}} + C_{i_\alpha}\Delta\alpha \\
&\quad + C_{i_\beta}\Delta\beta + C_{i_p}\Delta P + C_{i_q}\Delta Q + C_{i_r}\Delta R
\end{aligned}
$$

where $i = \{x, y, z, l, m, n\}$, $\Delta\alpha = \alpha - \alpha_0$, $\Delta\beta = \beta - \beta_0$, and so on where $\{\alpha_0, \beta_0, \ldots\}$ represent trim values.

Figure 7 shows the closed-loop behavior with the controller using the simplified representation of the database plotted along with the normal controller. The state trajectories with the two controllers are significantly different, but the simplified controller does stabilize an

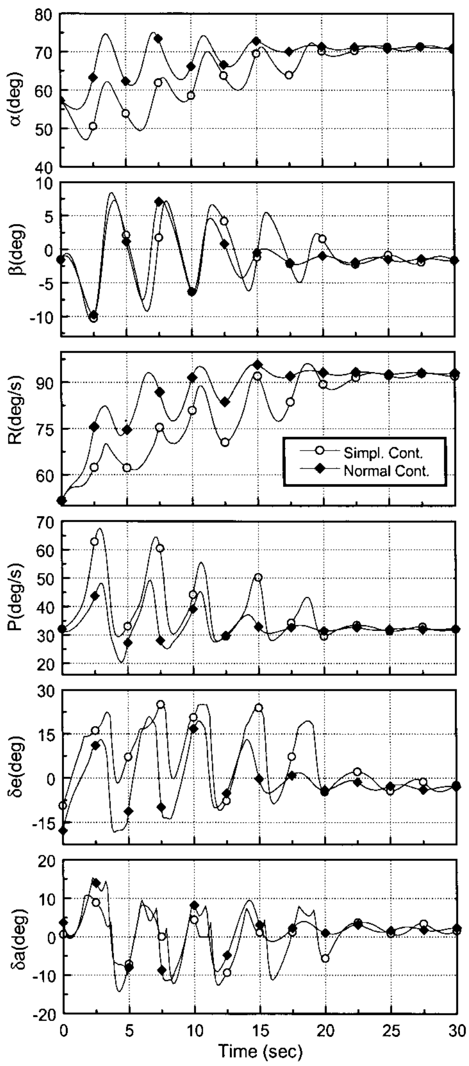

Fig. 7 Stabilization using the normal controller C_1 and the simplified controller with approximate database.

initial condition far removed from the equilibrium point in about the same time.

VII. Conclusions

A new two-controller methodology is described in this paper to control unstable aircraft configurations in poststall regimes. Feedback linearization technique has been used to design a stabilizing controller at high AOA. A novel technique is proposed for selecting outputs that ensure that the system has stable zero dynamics and hence results in an internally stable closed-loop system. The controller designed for stabilization of a spin equilibrium meets its objective of a large domain of attraction, as is established by extensive nonlinear six-degree-of-freedom simulations. With the synthetic pilot inputs designed for recovery and the stabilizing controller at low AOA, recovery from fairly complicated poststall conditions has been achieved. In short, the methodology suggested in this paper provides a viable scheme for recovering unstable aircraft from poststall conditions. Future studies will address issues related to robustness to variations in aerodynamic derivatives and mass and inertia of the aircraft.

Appendix: Constructive Procedure for Choosing a C Matrix to Place Transmission Zeros of a Linear System

For the system described by Eq. (6), let $\{n_1, n_2, \ldots, n_m\}$ be controllability indices. There exists a coordinate transformation[17] T such that

$$A^* = T^{-1}AT = \left[\begin{array}{c:c:c:c} A_1 & 0 & \cdots & 0 \\ \boldsymbol{e}_1^1 & \boldsymbol{e}_2^1 & \cdots & \boldsymbol{e}_m^1 \\ \hdashline 0 & A_2 & \cdots & 0 \\ \boldsymbol{e}_1^2 & \boldsymbol{e}_2^2 & \cdots & \boldsymbol{e}_m^2 \\ \hdashline \vdots & & & \\ \hdashline 0 & 0 & \cdots & A_m \\ \boldsymbol{e}_1^m & \boldsymbol{e}_2^m & \cdots & \boldsymbol{e}_m^m \end{array}\right]$$

$$B^* = T^{-1}B = \left[\begin{array}{c:c:c:c} \boldsymbol{b}_1 & 0 & \cdots & 0 \\ \hdashline 0 & \boldsymbol{b}_2 & \cdots & 0 \\ \hdashline \vdots & & & \vdots \\ \hdashline 0 & 0 & \cdots & \boldsymbol{b}_m \end{array}\right]$$

where each $\boldsymbol{e}_j^i$ is a row vector, A_i is $(n_i - 1) \times n_i$ matrix with the structure $A_i = [0 \mid I_{n_i - 1}]$, $I_{n_i - 1}$ is the $n_i - 1 \times n_i - 1$ identity matrix, and $\boldsymbol{b}_i$ is a column vector of dimension n_i with a structure $[0\ 0\ 0\ 0\ \cdots\ 1]^T$. Augment the set Z of desired transmission zero locations with m zeros and define $\tilde{Z} = [z_1, z_2, \ldots, z_{n-m}, 0, \ldots, 0]^T$. It is easy to find a matrix K^* such that $A^* - B^*K^*$ looks like

$$A^* - B^*K^* = \left[\begin{array}{c:c:c:c} A_1 & 0 & \cdots & 0 \\ h_1 & 0 & \cdots & 0 \\ \hdashline 0 & A_2 & \cdots & 0 \\ 0 & h_2 & \cdots & 0 \\ \hdashline \vdots & & & \\ \hdashline 0 & 0 & \cdots & A_m \\ 0 & 0 & \cdots & h_m \end{array}\right] \tag{A1}$$

where h_i is of the form $[0\ h_{i,1}\ \cdots\ h_{i,n_i-1}]$. This gain matrix K^* should be chosen to place the eigenvalues of $A^* - B^*K^*$ at $\tilde{Z}$. The form of $(A^* - B^*K^*)$ is such that a zero eigenvalue is placed by each input and thus it has an m-dimensional null space.

Claim: It is possible to construct a C such that the transmission zeros of system defined by Eqs. (6) and (7) are placed at Z.

Proof: Transmission zeros of Eqs. (6) and (7) can be computed as follows. Define a set of outputs:

$$\tilde{y} = \dot{y} = C\dot{x} = CAx + CBu$$

If CB is invertible, transmission zeros of Eqs. (6) and (7) are the nonzero eigenvalues of the matrix $(A - B(CB)^{-1}CA)$ (Refs. 10 and 11).

We shall determine a C such that

$$(A - B(CB)^{-1}CA) = A - BK \tag{A2}$$

where $A - BK$ has the eigenvalues at $\tilde{Z}$. This would ensure that nonzero eigenvalues of $(A - B(CB)^{-1}CA)$ are located at Z.

To satisfy the equality (A2), $(CB)^{-1}CA$ should equal K, and hence $CA = CBK$ and $C(A - BK) = 0$. Therefore we need to find a C such that

$$C(A - BK) = 0 \tag{A3}$$

However, we shall first determine C^* satisfying $C^*(A^* - B^*K^*) = 0$. Then we shall determine K, C to satisfy Eq. (A3) as described later.

In Eq. (A1), we already have the eigenvalues placed at $\tilde{Z}$, and knowing the structure of $A^* - B^*K^*$, it becomes easy to find a C^*. Consider an $m \times n$ matrix of the form

$$C^* = \begin{bmatrix} C_{1,1} & \cdots & C_{1,n_1} & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & C_{2,1} & \cdots & C_{2,n_2} & \cdots & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & C_{m,1} & \cdots & C_{m,n_m} \end{bmatrix}$$

For this C^* the product C^*B^* is given by

$$C^*B^* = \begin{bmatrix} C_{1,n_1} & 0 & \cdots & 0 \\ 0 & C_{2,n_2} & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & C_{m,n_m} \end{bmatrix}$$

This product is nonsingular if C_{i,n_i} is nonzero for $i \in \{1, \ldots, m\}$.

Now, if we want $C^*(A^* - B^*K^*) = 0$, then the following must be true for the elements in the ith row of C^*:

$$C_{i,1} = -h_{i,1} \cdot C_{i,n_i}$$
$$\vdots$$
$$C_{i,n_i-1} = -h_{i,n_i-1} \cdot C_{i,n_i}$$

Therefore, if the ith row of C^* has to be nontrivial, C_{i,n_i} must be nonzero for $i \in \{1, \ldots, m\}$. Thus C^* finally looks like

$$C^* = \begin{bmatrix} -C_{1,n_1} \,.\, h_1' & 0 & \cdots & 0 \\ 0 & -C_{2,n_2} \,.\, h_2' & \cdots & 0 \\ \vdots & & & \\ 0 & 0 & \cdots & -C_{m,n_m} \,.\, h_m' \end{bmatrix}$$

where $h_1' = [h_{i,1} \quad \cdots \quad h_{i,n_i-1} \quad -1]$.

Now, define $C = C^*T^{-1}$ and $K = K^*T^{-1}$. For this choice of C and K,

$$\begin{aligned} C(A - BK) &= C^*T^{-1}(A - BK^*T^{-1}) \\ &= C^*T^{-1}(TA^*T^{-1} - TB^*K^*T^{-1}) \\ &= C^*(A^* - B^*K^*)T^{-1} = 0 \end{aligned}$$

Hence we have obtained a C such that $C(A - BK) = 0$ and $\text{eig}(A - BK) = \text{eig}(A^* - B^*K^*) = \tilde{Z}$, so by the statements at the beginning of the proof, we have placed the transmission zeros at Z by choosing a C matrix.

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